Forest Resource Economics

Unit 1: Introduction

Scope of Forest Resource Economics

Economics is the science which deals with allocating scarce resources among competing means to satisfy the wants of mankind. Forest could be regarded as the natural use of any and all land that was not needed for agriculture and grazing. It is a renewable or flow resource in which the flow is significantly affected by human action and the reversibility of a decrease in flow is characterized by a critical zone. Forest resources are the product from the forest lands that have great value to human beings.

Forest economics is an applied field of economics which deals more with problems in application than with theory. It is the application of microeconomic principles to forestry's managerial and decision making problems. The natural resource personnel are generally biologically oriented. Solution to problems is resource biased. Microeconomics focus attention on the concept of opportunity cost and raises questions like whether the benefit to the resource is worth the cost to the resource owner or the nation.

Forest economics is concerned with the management of forest resources, their regeneration, protection, conservation and discreet harvesting consistent with judicious utilization: conversion, distribution and consumption. In other words forest economics deals with economic problems involved in buying, owning, selling, taxing and managing forest land whether used for production of water, wildlife, wood or any other product. Within the forestry profession economics generally seeks the answer to 'why' of a problem rather than 'how'. A detailed knowledge of the peculiarities of forestry is required if economic concepts and principles are to be appropriately applied in the field of forestry.

The field of forest economics can be divided into two categories:
1. Forest industry economics which deals with the study of manufacturing logs, lumber etc.
2. Forest management economics which deals with all economic problems concerning the growing of forests and producing all products like timber, wildlife, water, outdoor recreation and forage.

A forest economist deals with both the above categories but more so with the second.

Relevance of Economic Theories in Nepalese Forest Resource Management

Forest economics is the application of economic theories to forestry problems as related to economic growth and human welfare. Forest is our heritage and their utilisation should be based on scientific considerations. Economic theories and principles helps a forester to make better recommendation and judgements so that the forestry sector contributes its best to the economic well being of society.

In Nepal very little attention has so far been paid to economic theories and tools in solving forestry problems. Scarcity of forest resources increase the important role that economic theories can play in sound forest management and planning. Some important concepts and techniques of economic theory that help a forester in his profession are discussed below:

1. **Critical analysis approach:** Economics establish cause and effect relationship of a phenomenon. The concept leads the foresters to view with a more logical approach the natural resource management problem.

2. **Emphasis on the relevancy of opportunity cost:** This entails the problem of scarcity and choice by measuring the cost of a unit of any commodity in terms of other commodities that would have been obtained.

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3. Use of models: This requires examination of variables, identification and interrelationship of key variables. Useful in planning the development of natural resources.

4. A framework for the analysis of field problems: Marginal analysis is a major tool in microeconomic theory. This principle finds wide applicability in forestry.

5. Guidance to government: The microeconomic theory holds that social welfare can be optimised only when the price system works perfectly. In the absence of this government intervention to introduce a recreational facility may cause misallocation of resources. Economic analysis suggests the suitable steps.

6. The concept of production possibility curve: This helps a forester to understand better the resource allocation problems.

7. The value added concept: This concept applies intimately to forestry. For optimization of socio-economic benefits from forests scientific utilization of forest products with due care to their regeneration and perpetuation of resource bases and environment should be an integral part of forestry.

8. Aid to capital budgeting and resource allocation: Economics helps to critically evaluate the available alternatives through meaningful socio-benefit-cost analysis and aids the decision-maker in choosing an alternative.

9. Economic growth and development: An understanding of economic growth and development is helpful to the foresters. It shows how forestry should be practiced for rural development. It also helps him to understand the contribution of forest sector to GDP/GNP.

10. Forestry as a social service: Forestry is mainly a social service. Therefore it is not possible to assess its projects purely on the basis of commercial benefit-cost analysis. Its benefit and costs must take into account the various relevant factors which effect national welfare, environment, resources and security. Economic theory helps Nepalese foresters appreciate the need and necessity of measuring the environmental impacts of the flora and fauna. It helps to evolve new criteria for assessment of various aspects of forestry's role, to provide factors at present beyond the scope of traditional benefit-cost analysis.

Some other important specific decisions to be made by foresters where economic analysis can be helpful are:

i) Allotment of forest land to protection forestry, productive or commercial forestry, social forestry etc.

ii) Assessment of the extent of employment potential of forestry.

iii) The product mixed to be planned in the utilisation pattern of wood.

iv) Determination of rotation

v) The factors governing the pricing policy for supply of raw materials to forest based industries.

vi) The policy regarding the form in which to export forest product.

vii) The policy regarding the form in which forest products should be marketed or supplied to the people.

viii) Analysis of people's involvement and participation in forest and wildlife management and its impact on policy and decision.

ix) Protection of the endangered species.

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Economic History of Forest Management and Conservation

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Unit 2: The Firm in Market Economy

Cost Function

The functional relationship between costs and output is called cost function. Cost functions are derived function. They are derived from production function. The production function of a firm and the prices for its inputs determine the firm's cost function.

Economic theory distinguishes between short-run costs and long-run costs. Short-run costs are the costs over a period of during which some factors of production (usually capital equipment and management) are fixed. The long-run costs are the costs over a period long enough to permit the change of all factors of production. In the long-run all factors become variable.

Both in the short-run and in the long-run, total cost is a multivariable function. Symbolically the long-run production function can be written as:

$$C = f(X, T, P_f)$$

and the short-run cost function as:

$$C = f(X, T, P_f, \overline{K})$$

where C = total cost, X = output, T = technology, P_f = prices of factors, $\overline{K}$ = fixed factors

Graphically, costs are shown on two-dimensional diagrams. Such curves imply that cost is a function of output, all other factors remaining the same. If these factors do change, their effect on cost is shown graphically by a shift of the cost curve. But mathematically there is no difference between the various determinants of cost.

Practical examples of some cost functions are given below:
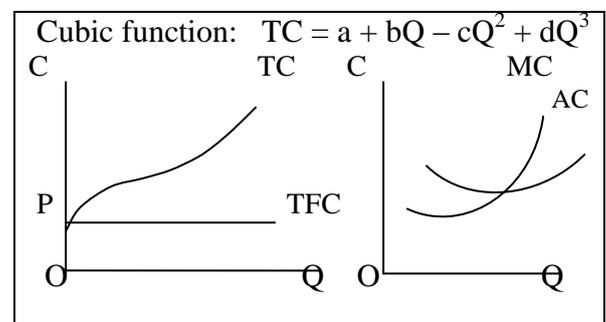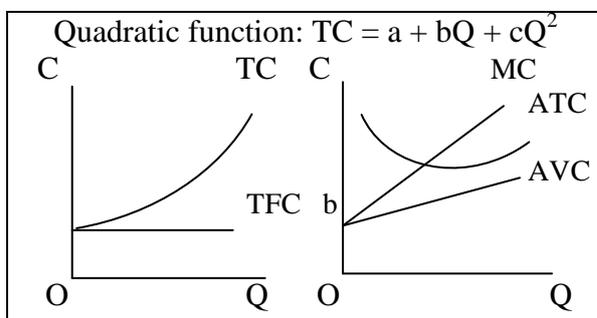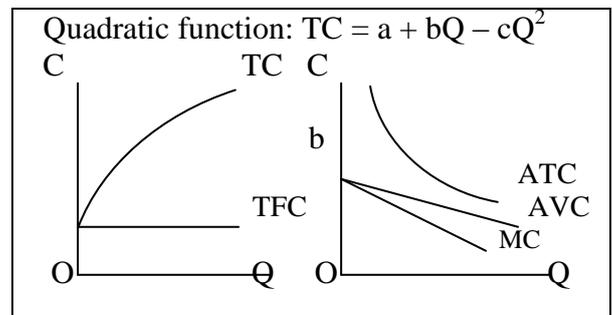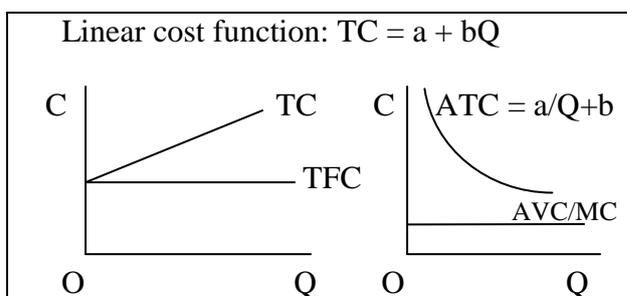
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Input Demand and Supply

Demand for Inputs

The demand for inputs or factors differs from that of the goods. The demand for goods is direct but factor demand is derived. The factors of production are demanded only because they help in the production process. The elasticity of demand for the input is determined by three forces:

1. **Quantity of the factor:** The elasticity of demand of an input depends on its ratio with the other factors. If it forms only a small proportion among the various factors used, its price change will have little effect on its demand.

2. **Elasticity of demand for the final product:** Because the demand for inputs is derived demand, its demand depends upon the elasticity of demand for the good produced by it. If the demand for the good it produces is inelastic or less elastic, its own demand will also be inelastic or less elastic and vice versa.

3. **Possibility of factor substitution:** If a factor can be used in more than one ways, i.e. if it is substitutable, its demand will be more elastic. Because when its price falls, it can be used in place of other inputs whose price remains constant.

Therefore we can conclude that the demand for the inputs is determined by its marginal revenue productivity (MRP). When more and more factors of the same type are put to work, their MRP declines. It is also advisable for any firm to apply the factors up to the point where factor price is equal to its MRP. This means there is an inverse relationship between the price of a factor and its demand and therefore the demand curve of a factor is downward-sloping from left to right. To conclude, MRP curve is demand curve for a factor of an individual firm. Under perfect competition, if all the firms produce with identical costs the demand from the industry is the aggregate of the demand from all the firms. In other words the summation of MRP curves of all the firms is the demand curve of the industry. The derivation of the factor demand curve of an industry can be explained diagrammatically.
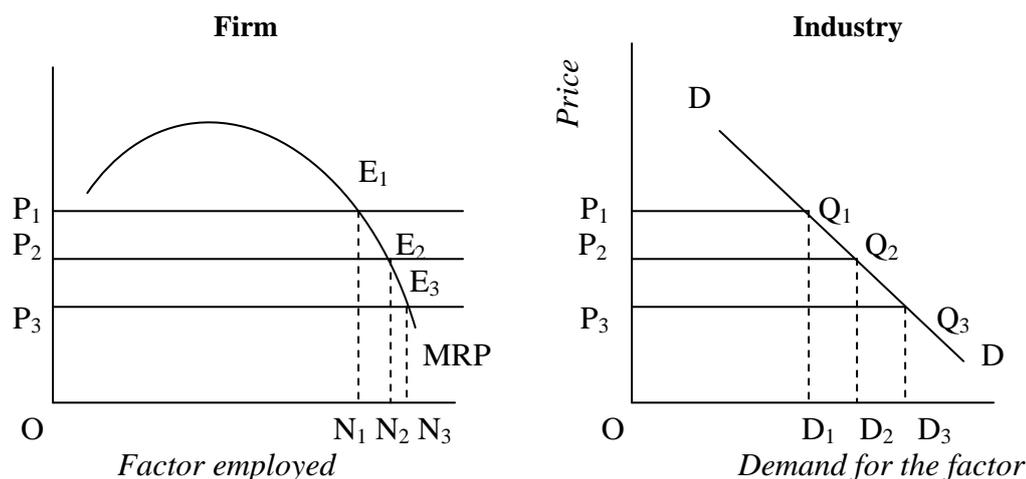
In the figure, MRP is the demand curve of the firm for the factor. When the price of the factor is OP_1, equilibrium point of the firm is E_1, where ON_1 factor units are demanded. Here MRP = Price = N_1E_1. Now suppose there are 100 firms in the industry; the industry demand for the factor is 100 X ON_1 which is shown by the point Q_1 in the right hand side of the figure. Here demand is OD_1. When the factor price falls to OP_2, firm's demand for it is ON_2 and the total demand from industry is OD_2 (ON_2 X 100). Similarly, when price falls to OP_3, the firm's demand increases to ON_3. The demand of the industry is OD_3 (ON_3 X 100). The equilibrium point of the industry is Q_3. When the points Q_1, Q_2, Q_3 are joined; we get a demand curve DD

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for the industry. The slope of the demand curve is downward i.e. there is an inverse relationship between the price of a factor and its demand.

Supply of Inputs

The supply of inputs is very much different from that of goods. The supply of goods increases with the rise in their prices and vice versa. But the supply of factors of production is not so simple. Price is not the only governing factor on the supply side of the factor units. There are so many other things affecting it. The supply of some factors often remains the same or even falls with the rise in their prices. For example, the supply curve of labour is backward sloping, i.e. its supply falls with an increase in its price. Similarly, for a single firm the supply of land can be increased with increase in its price. But for the economy as a whole the supply is fixed. The supply of capital also does not depend only on the rate of interest but also on levels of income as Keynes believed.

But for simplicity we believe that the market demand curve of a factor is upward sloping. In ordinary cases, the proposition of a rising supply curve is true.

Determination of Equilibrium Price for an Input

Modern theorists believe that factor price is fixed at the level where the demand and supply equate each other. This is called the equilibrium price of factor. The determination of the equilibrium price is shown in the fig given below.

Demand and supply of the factor
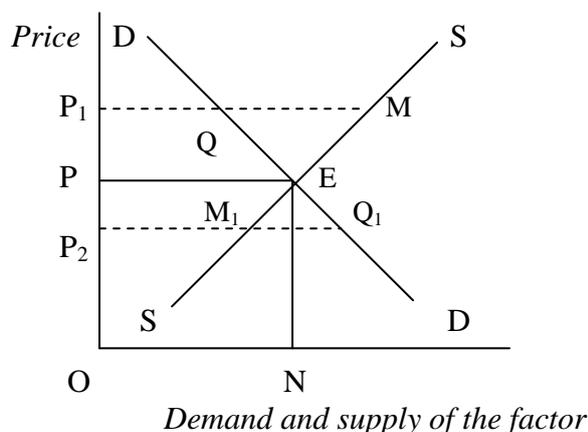
In the figure DD is the demand curve and SS is the supply curve of a factor for industry. E is the point of equilibrium. The factor price is determined at OP. If the factor price rises to OP_1, QM factor units will be idle. It will bring down the price to OP. Similarly, if price falls to OP_2, there will be excess demand. The excess demand bid up the price to OP.

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Economic Method of Multiple Production

When enterprises are combined for multiple-product management, the combining is termed as integration. There are two types of integration – vertical integration and horizontal integration. When enterprises representing successive links in the economic chain of production in a firm are combined it is said that the firm is vertically integrated, for example firm engaged in growing trees and also logging them. When enterprises that use some of the same inputs are combined within a firm, the firm is said to be horizontally integrated, for example firm producing lumber and pulp. A single firm can both be vertically and horizontally integrated.

Vertically Related Products

Stumpage, logs and lumber are vertically related products of an integrated firm. The usual practice is that the output of the first product in the production chain, that is thought maximize profits, is planned to be produced. For example, the firm may decide to produce X units of lumber because it maximises the profits of the firm. For this output of lumber Y units of logs may be required. Therefore the logging enterprise will plan to harvest Z units of forest area to meet this requirement.

Under the constraints of imperfectly developed markets and government regulations this may be the only means of maximizing profit for the firm. However, if there are perfect competitive markets for each product the appropriate policy for the firm is to maximize profit of each enterprise by producing at the point where his marginal revenue equals his marginal cost.

Horizontally Related Products/ Joint Production

As long as the two products are produced by entirely separate processes no special problems arise. Problems arise when the same production facility is used to produce two or more products. This is the case of joint production. Joint production can be of two types – production in **technically fixed proportions** (wheat and straw, beef and hides etc.) and in **technically variable proportions** (sawlogs and pulpwood, herbs and stumps etc.)

Technically Fixed Proportion

With technically fixed proportions, the product combinations hold a constant ratio to each other. If the combinations are plotted on a diagram it will take a shape of a straight line passing through the origin as shown in fig.1 by OD.

Fig. 1

Joint production with technically fixed proportions can always be reduced to the single product case. If, for example, in a particular sawmill one ton of pulpwood chips can be produced with every 1400 board feet of sawnwood, the problem of joint production can be solved by treating this as a "combination" product. The total cost and revenue picture might appear as in fig.2, where single cost and revenue curves are used while the horizontal axis shows quantities of both lumber and chips at all relevant outputs. The profit maximizing solution is obtained exactly as before, and a full set of total cost curves can be derived. But average costs cannot be derived. An average cost for lumber or for pulpwood chips cannot be calculated. It does not exist. Whenever two products are produced jointly there is
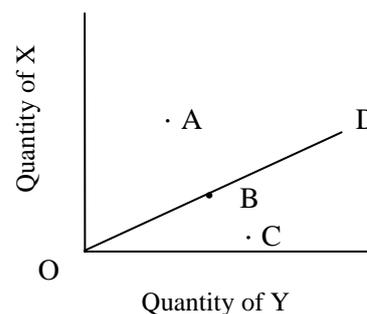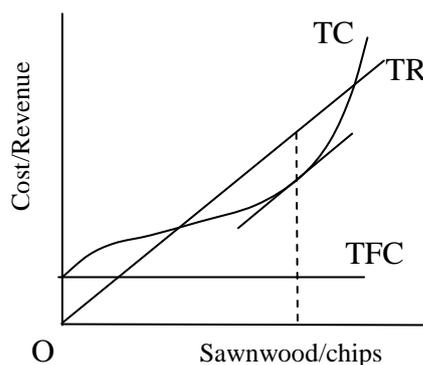
Fig. 2

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simply no way of determining the average cost for either product.

In purely competitive situations (where the prices of both products are given to the producer) no particular treatment of the firm's demand is necessary. But, from the standpoint of industry, market analysis, and pricing the two jointly produced products may be quite different. Determination of price of jointly produced goods is shown in fig.3. In the fig. D_s represent demand for sawlogs and D_c shows demand for chips. Then the vertical summation of these curves represented by D_{sc} in the fig. indicates industry's total demand curve for sawlogs and chips. If the supply curve for the combination product is represented by S_{sc}, the combination price is determined at P_{sc}. Accordingly the individual price for sawlogs and chips is determined at P_s and P_c respectively as shown in the fig.
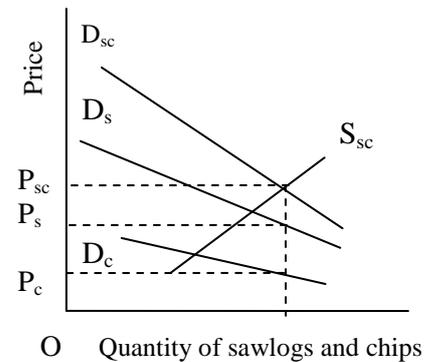
Fig. 3

Technically Variable Proportion

In this case, although joint use of some productive factor or service is involved in production, the proportion in which the products can be produced may be varied. For example a given hectare of forest can be made to yield sawtimber and pulpwood in varying proportions. With given inputs output of one product can be increased by reducing the output of other product/s. This is shown in fig.1 assuming that it is possible to produce any combination among A, B, C etc. with given inputs. If we join all these possible combinations we get a curve as shown in fig.4 called **production possibility curve** (PPC). With increase in inputs i.e. the cost of production PPC shifts to the right hand side, showing more outputs as shown in the fig. Now if the products are sold in the market firm will get revenue. Different combinations of products will give different level of revenue to the producer. Joining all combinations of the products which give equal revenues we get a curve called **isorevenue curve**. Under perfect competition as price and therefore price ratio of the products is fixed, isorevenue curves will be straight line and parallel to each other as shown in fig.5.
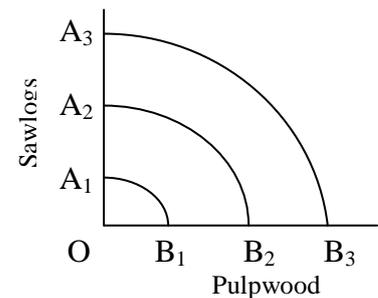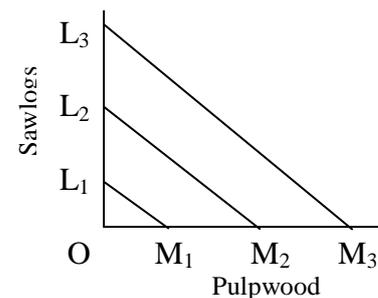
Combining these isorevenue curves with previous set of PPCs we get a diagram as in fig.6. Each tangency point in the fig. is the point where revenue is maximum for any particular cost outlay. If we join all these points of tangency we get a curve called **expansion path** and a rational producer will always operate in such a manner as to produce a combination which lie on this curve. Now the problem is which combination is the optimum?
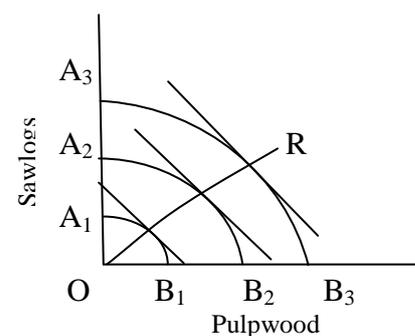
The complete solution is shown in fig.7, which is a combination of fig.6 with the resulting total cost / total revenue curves. The profit maximizing combination can be

Fig. 4

Fig. 5

Fig. 6

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read from point on the expansion path that falls directly below the point where the slopes of the total cost and total revenue curves are equal.

In this case also average costs for either product can not be found out. An average cost of sawlog production or pulpwood production does not exist. Conceivably one could construct a pseudo-average cost diagram by employing a device similar to that of fig.7.

But marginal costs do exist and can be determined for either product. Because the marginal cost equation for either product will contain the output of the jointly produced product as a variable, the two marginal cost curves must be solved simultaneously. Algebraically this can be calculated as below:

Given the total cost as a function of the output of two products, X and Y,

$$TC = f(X,Y)$$

then the marginal costs of X and Y become, respectively,

$$MC_X = \partial(TC)/\partial X \quad \text{and} \quad MC_Y = \partial(TC)/\partial Y$$

Fig. 7

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Linear Programming
Introduction
Linear programming is a mathematical optimization technique designed to assist an organization in allocating its scares resources for the achievement of its goal. Here *optimization* means a method which attempts to maximize or minimize the objective e.g. maximize profits or minimize costs. The term *linear* implies that the relationship involved must be linear. The term *programming* means taking decisions systematically. In short, linear programming is an optimization technique under certain limitations or constraints. The constraints should be represented by linear inequalities.

L.P. was developed by a mathematician named George B. Dantzig in 1947. His original application was in the area of military planning for U.S. Air Force.

Basic Concepts
Objective function: The quantity to be minimised or maximised is expressed through a function called *objective function* or *criteria function.*

Constraints: The objective functions of minimisation or maximisation problem are subject to certain constraints which prevent the solutions being infinitely large or small. This limiting condition arises due to limited resource or technological conditions.

Choice/decision variables: These are the numbers which are to be chosen so as to minimise or maximise) the objective function satisfying all the constraints.

Feasible and optimum feasible solution: Feasible solutions are those that can be achieved with given resources under given constraints. Optimum feasible solution is the particular feasible solution, which yield the optimum value.

Basic Assumptions of L.P.
Linearity: This implies a linear relationship between input and output. In other words, linear programming solutions are based on the assumption of constant returns to a factor.

Proportionality: This property arises from the assumption of linearity. Proportionality implies that the flow of resources into an activity is proportional to the level of activity. For example, if one unit of potatoes requires 1 acre of land and 2 hours of labour, then 2 units of potatoes require 2 acre of land and 4 hours of labour.

Independence and additivity: The variables and their quantitative specification are independent of other variables. Also the total amount of resources used by all activities must equal the sum of the resources used by the individual activities.

Divisibility: This means that each activity can take any fractional value in production process.

A linear objective function: The proportionality and additivity properties hold for the objective function as well as for the constraints. All coefficients in both the objective function and the constraints of a linear programming model are assumed to be constant. Because of this property, the linear programming approach is especially well suited to purely competitive market situations wherein the prices of both outputs and inputs are assumed to be unaffected by the actions of any single individual or activity.

Non-negativity: This is necessary to guarantee the real economic interpretability of activity levels. The variables with which we deal cannot be negative.

Formulation of Linear Programming Problem
The formulation of L.P. problem requires defining the decision variables in equations say x_1, x_2, ……………, x_n. The term formulation implies the translation of a real world problem into a format of mathematical equations. The basic structure of a L.P. problem is either to maximise or to minimise an objective function subject to a set constraints. The objective

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function is a mathematical representation of overall goal stated in terms of decision variables x_n. The set of constraints, expressed in terms of x_n, represents the conditions which must be satisfied in determining levels for x_n.

To understand the formulation of L.P. problem, we take an example below.

Example 1: A company manufactures two types of products A and B. Product A requires 2 hours to make, and the product B requires 4 hours. The company has only 800 work hours to use in manufacturing each day, and the packing department can package only 300 units of product per day. If the company sells the product A for Rs. 30 and product B for Rs. 40, how many of each type of product should it produce per day to maximise its sales?

Step 1: Identify the choice/decision variables.

Let x_1 =   number of units of product A produced per day
           x_2 =   number of units of product B produced per day
           Z   =   total sales of the company. Here x_1 and x_2 are decision variables.

Step 2: Define the objective function. The objective function is:

$$\text{Maximise } Z = 30x_1 + 40x_2$$

The above equation states that the total sales are equal to Rs. 30 times of x_1 and 40 times of x_2.

Step 3: Formulate the constraints.

The first constraint relates the availability of total work hours to use in manufacturing each day. This can be expressed as:

$$2x_1 + 4x_2 \leq 800$$

This linear inequality states that the manufacturing time of 2 hours for product A times the number of units produced plus the manufacturing time of 4 hours for product B times the number of units produced should not exceed 800 work hours.

The second constraint relates the packaging capacity of the company each day, this can be expressed as:

$$x_1 + x_2 \leq 300$$

Step 3: Formulation of non-negativity constraints.

In any L.P. formulation there are constraints which restrict the choice variables to be non-negative, are known as non-negativity constraints and stated as:

$$x_1, x_2 \geq 0$$

Step 4: State the problem. The final form of formulation is as follows:

     Maximise $Z = 30x_1 + 40x_2$ (objective function)
     Subject to $2x_1 + 4x_2 \leq 800$ (manufacturing time constraint)
               $x_1 + x_2 \leq 300$ (packaging constraint)
               $x_1, x_2 \geq 0$ (non-negativity constraint)

Graphical Method

Maximisation Problem: *Step 1* − express the statement of problem in mathematical formulation. Referring above example the mathematical formulation is:

     Maximise $Z = 30x_1 + 40x_2$
     Subject to $2x_1 + 4x_2 \leq 800$
                 $x_1 + x_2 \leq 300$
                 $x_1, x_2 \geq 0$

Step 2 − Plot the constraints of the problem in the graph paper as shown in the fig. For this draw the line $2x_1 + 4x_2 = 800$ by locating and then joining its two terminal points. To find the terminal points first put $x_2 = 0$ and then $x_1 = 0$ in the above equation. This will give the points (400, 0) and (0, 200). Similarly draw the line $x_1 + x_2 = 300$.
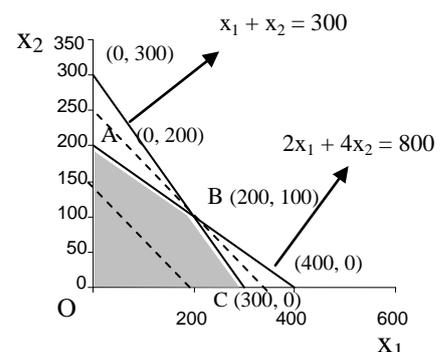
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Step 3 – Determine the feasible region by shading the area as shown in the fig. The feasible region OABC is the collection of all possible sets if solutions that meet the given constraint.
Step 4 – Maximise sales subject to above shaded area. This can be done in two ways – search approach (corner point) or iso-sales line approach. The search approach needs the evaluation of each corner points on the feasible region by solving the equation of the objective function:

$$Z = 30x_1 + 40x_2$$

Corner points	Feasible solution of product mix (x_1, x_2)	Total sales (Rs.) $Z = 30x_1 + 40x_2$
O	(0, 0)	0
A	(0, 200)	8000
B	(200, 100)	10000 (maximum)
C	(300, 0)	9000

From the above table, it is clear that the sales are maximum at B when $x_1 = 200$ and $x_2 = 100$ units. This is the optimum solution out of several feasible solutions. This optimum point can also be found out by using iso-sales line approach i.e. by drawing the straight line for objective function and moving upwards (or downwards) slowly parallel to the line till it touches the last point on feasible region. The iso-sales line refers to the fact that any combination of points on the same line produces the same sales. For drawing the line for objective function, the terminal values need to be calculated as we did in the case of terminal values in inequalities.
(Point B can be determined by solving simultaneous equations of the two lines which intersect at the point B)

Minimisation Problem:

Example 2: Minimize $C = 6x_1 + 8x_2$
         S.t. $4x_1 + 3x_2 \geq 120$
             $3x_1 + 6x_2 \geq 120$
             $x_1, x_2 \geq 0$

Sol.: The objective function and constraints are graphed in the fig. the shaded area is the feasible region. All the feasible solutions lie in the feasible region. The corner point M is obtained by solving the equations $4x_1 + 3x_2 = 120$ and $3x_1 + 6x_2 = 120$ simultaneously.
The computation of total cost corresponding to the points A, M and B are as follows:

Corner points (x_1, x_2)	Total cost $C = 6x_1 + 8x_2$
A (0, 40)	320
M (24, 8)	208 (minimum)
B (40, 0)	240

At point M, the objective function is tangent to the feasible region. Hence the point M is the optimum solution.
Thus C is minimised at Rs. 208 when $x_1 = 24$ and $x_2 = 8$.
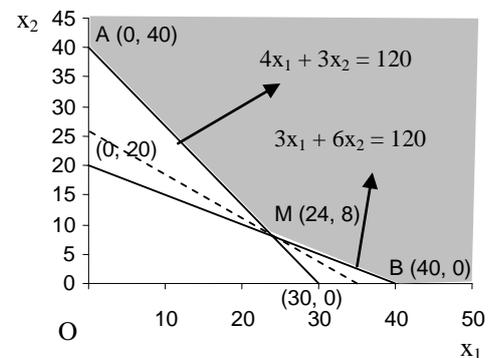
General Formulation of L. P. Problem

Max. $Z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$
s.t. $a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n \leq b_1$
     $a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n \leq b_2$
     $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

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$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n \leq b_m$$
$$x_1, x_2, \ldots\ldots\ldots \geq 0$$

Using sigma notation:

$$\text{Max.} \quad Z = \sum_j c_j x_j$$

$$\text{s.t.} \quad \sum_j a_{ij}x_j \leq b_i$$

and $\quad x_j \geq 0$

where $i = 1, 2, \ldots\ldots\ldots, m$ and $j = 1, 2, \ldots\ldots\ldots, n$

Limitations of L. P.
1. There is no guaranty that the LP will give integer valued solutions. For example the solution may be 2.4 units of houses and 4.7 units of cars which do not exist in practice.
2. LP does not allow for uncertainty. It is assumed that the constraints and requirements are known. But in real problem these functions may not be known.
3. Sometimes constraints in real business problems may not be linearly related. Hence the coefficients assumed to be constants may not hold good.

Some Complications in LP
Infeasibility: Infeasibility means there is no feasible solution which satisfies all the constraints. This is illustrated by an example below

$$\text{Max.} \quad P = 2x_1 + 4x_2$$
$$\text{s.t.} \quad 4x_1 + 6x_2 \leq 36$$
$$2x_1 + 4x_2 \leq 20$$
$$x_1 \geq 10$$
$$x_2 \geq 10$$
$$x_1, x_2 \geq 0$$
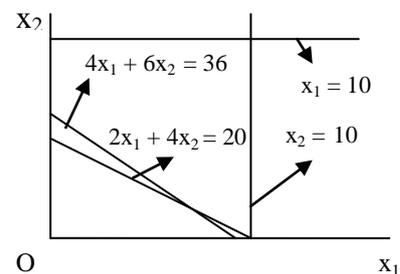
In this LP problem, it is clear that all the constraints cannot be made at the same time. This means, there does not exist the feasible region. Therefore there is no optimal solution to the problem. This is the condition of infeasibility in linear programming.

Unboundedness: A LP problem is unbounded if the solution can be made infinitely large without violating any of the constraints in the problem. In this case there will not be finite solution. This can be illustrated by the following example:

$$\text{Max.} \quad P = 2x_1 + 4x_2$$
$$\text{s.t.} \quad x_1 \geq 10$$
$$x_2 \geq 10$$
$$x_1, x_2 \geq 0$$
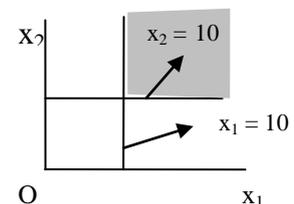
Degeneracy: In some cases, a constraint does not form a unique boundary of the feasible space and has no impact on the solution of the problem. Such a constraint is called redundant constraint because of its removal would not alter the nature of the feasible region. Degeneracy occurs because of redundant constraint. Sometimes the redundant constraint can be identified in the beginning if it is so, the redundant constraint can be removed from the problem that saves the computational time as in the following example:

$$\text{Max} \quad P = 2x_1 + 4x_2$$
$$\text{s.t.} \quad 4x_1 + 6x_2 \leq 36$$
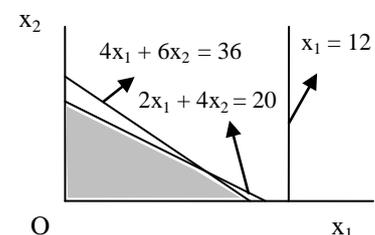
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$$2x_1 + 4x_2 \leq 20$$
$$x_1 \leq 12$$
$$x_1, x_2 \geq 0$$

Here the third constraint is redundant as it does not affect the feasible region. When the first two constraints are met, the third constraint will be automatically fulfilled.

Multiple solutions: There may be more than one optimum solutions of a LP problem. In such case two or more corner points will have the same optimum profits or cost. This happens when the slope of the objective function coincides with the slope of one of the constraint equations. This is illustrated below.

Example3: A small firm manufactures two products A and B. the combined number of A and B that it can handle is 24. Each product B takes 1 hour of labour and each product A takes a half hour. The total number of hours of labour available per day is 16. If the profit on the product A is Rs. 1 and that on product B is Rs. 2, how many of each product should be produced per daily to maximise profit?

Sol.: let x1 and x2 be the number of products A and B produced daily respectively. Then the given problem can be formulated as:

Maximise profit $\quad P = 2x_1 + x_2$
s.t. $\quad x_1 + \frac{1}{2} x_2 \leq 16$
$\quad x_1 + x_2 \leq 24$
$\quad x_1, x_2 \geq 0$
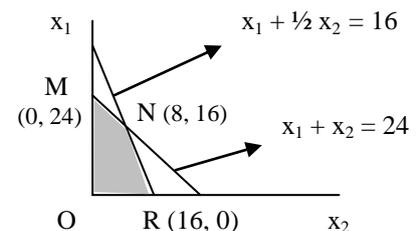
The corners of the polygon OMNR are O (0, 0), M (0, 24), N (8, 16), R (16, 0). The corner N (8, 16) can be found by solving $x_1 + \frac{1}{2} x_2 = 16$ and $x_1 + x_2 = 24$ simultaneously. Testing the objective function on corners, we obtain,

Corner point (x_1, x_2)	$P = 2x_1 + x_2$
O (0, 0)	0
M (0, 24)	24
N (16, 0)	32 (maximum)
R (8, 16)	32 (maximum)

Thus the maximum value of the function occurs either at $x_1 = 16$, $x_2 = 0$ or $x_1 = 8$, $x_2 = 16$, which means that the profit will be maximised at any point on the line joining (8, 16) and (16, 0). For example the point (12, 8) is on this line, and the value of P at this point is P = 2*12 + 8 = 32.

Exercise

1. A petroleum refiner sells two products, gasoline and fuel oil. Net profit from gasoline is Rs. 160 per barrel and from the fuel oil Rs. 8 per barrel. There are two constraints. Storage tank capacity is limited to 5000 gallons for gasoline and 12000 gallons for fuel oil. The refining plant has a maximum capacity and each gallon of fuel oil 2 units. Determine graphically the most profitable product mix and estimate profit. *Take 1 barrel = 31.5 gallons.* [*Ans. Max. P = Rs. 40320000, gasoline = 5000 gallons, fuel oil = 6000 gallons.*]

2. Two factories produce three different types of kitchen appliances. The table below summarizes the production capacity, the number of each type of appliance ordered, and the daily operating costs for the factories. How many days should each factory operate to fill the orders at minimum cost?

	Factory1	Factory2	Number ordered
Appliance 1	80 per day	20 per day	1600
Appliance 2	10 per day	10 per day	500
Appliance 3	20 per day	70 per day	2000
Daily cost	Rs. 1,00,000	Rs. 2,00,000	

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[*Ans. Min. C = Rs. 70,00,000, factory 1 = 30 days, factory 2 = 20 days.*]

3. An electric company produces two products A and B. the weekly production cannot exceed 25 for A and 35 for B. the company employs a total of 60 workers. Product A requires 2 man-weeks of labour where B requires only one. Profit margin on A is Rs. 60 and on B is Rs. 40. Find maximum profit assuming that all that is produced can be sold. [*Ans. Max P = Rs. 2150, A = 12.5 units, B = 35 units.*]

4. Two foods contain only protein, carbohydrate and fat. Food F_1 costs Rs. 100 per kg. And contains 30% protein and 50% carbohydrate. Food F_2 costs Rs. 150 per kg And contains 20% protein and 75% carbohydrate. What combination of these foods provides at least 1 kg of protein, 2.5 kg of carbohydrate and 250 grams of fat at the lowest cost? [*Ans. C_{min} = Rs. 125, F_1 = 1.25 kg, F_2 = 0*]

5. A company produces two types of toy model cars T_1 and T_2. Three machines M_1, M_2 and M_3 are required to produce each type. Each T_1 requires 7 minutes of process time on machine M_1, 4 minutes on M_2 and 10 minutes on M_3. A model T_2 requires 4 minutes on M_1, 15 minutes on M_2 and 16 minutes on M_3. Each day there are 420 minutes available on M_1, 600 minutes on M_2 and 840 minutes on M_3. The variable costs are Rs. 10 per unit for T_1 and Rs. 14 per unit for T_2. Each toy T_1 is sold at Rs. 18 and each toy T_2 is sold at Rs. 24. The company's fixed costs are Rs. 180 per day. Using the graphic method, determine the daily production rates for the two models which will maximise profit? [*Ans. P_{max} = Rs. 426, T_1 = 47, T_2 = 23*]

6. A factory owner wishes to use a combination of radio and TV advertisements for his new product. Research has shown that each 1-minute spot on TV reaches 90,000 people and each 1-minute spot on radio reaches 6,000. The owner feels he must reach at least 2.16 million people, and he must buy a total of at least 80 minutes of advertisements. How many minutes of each medium should be used to minimise costs if TV costs Rs. 500 per minute and radio costs Rs. 100 per minute? [*Ans. Min. C = Rs. 16000, radio = 60 minutes, TV = 20 minutes*]

Simplex Method

Finding the extreme point: The optimal solution of a linear program is to be found among the extreme points. Given a graphable feasible region F, it is a simple task to find its extreme points. To find it for the nongraphable n-variable case, let us re-examine the extreme points of the 2-variable case, which is given below, for possible clues.

Example4: A business firm produces two lines of product I and II, with a plant that consists of three production departments: cutting, mixing and packaging. The equipment in each department can be used for 8 hr a day. Product I is first cut then packaged. Each ton of this product uses up ½ hr of the cutting capacity and 1/3 hr of the packaging capacity. Product II is first mixed, then packaged. Each ton of this product uses up 1 hr of the mixing capacity and 2/3 hr of the packaging capacity. Finally, product I and II can be sold at prices of Rs. 80 and Rs. 60 per ton, respectively, but after deducting the variable costs incurred, they yield on a net basis Rs. 40 and Rs. 30 per ton. These latter amounts may be considered as gross-profit figures. What output combination should the firm choose in order to maximise the total gross profit?

Sol.: Let x_1 and x_2 be the output of product I and II respectively, then the translated problem will be:

Max. Profit, $P = 40 x_1 + 30 x_2$
s.t. $x_1 \leq 16$ [cutting constraint]
      $x_2 \leq 8$ [mixing constraint]
(1) $x_1 + 2x_2 \leq 24$ [packaging constraint]
      $x_1, x_2 \geq 0$
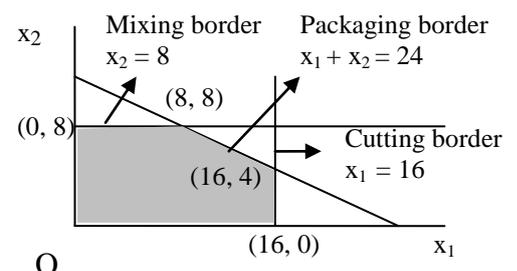
The feasible region of this problem is shown graphically in the above fig. by shaded area.

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The extreme points in the fig. fall into three major types. The first type consists of those which occur at the intersection of two constraint borders; examples of these are found in the points (8, 8) and (16, 4). While such points do satisfy two of the constraints exactly, the remaining constraint is inexactly satisfied. The point (16, 4), for instance, exactly satisfies the cutting and packaging constraints but not the mixing constraint, because the point lies below the mixing border. With the inexact satisfaction of some constraint, there must be an underutilisation of capacity. That is a slack in capacity utilisation (or a surplus in minimisation problem) will develop.

Extreme points of the second type, exemplified by (0, 8) an (16, 0), occur where a constraint border intersects an axis. Because they are located on one constraint border only, such points can satisfy only one constraint exactly. This means there will now develop slacks in two constraints.

Last, as the third type of extreme point, we have the point of origin (0, 0), which satisfies none of the constraints exactly. The (0, 0) extreme point, however is only found in a maximisation problem, for the feasible region of a minimisation problem normally excludes the point of origin.

This means in the present example each and every extreme point will involve a slack in at least one of the constraints. Furthermore, as can be seen in the fig, the specific value of slacks at each extreme point is easily calculable. Therefore, when we get the optimal solution, we will have not only the optimal values of x_1 and x_2 but also the optimal values of slacks. Now to find slacks explicitly, let us denote the slack in the ith constraint by the symbol s_i. The si represents slack variables (or surplus variables in minimisation problem). This will enable us to transform each constraint inequality into a strict equality. More importantly, it will lead us to an algebraic method of finding the extreme points of feasible region.

By adding a slack variable to each constraint and appropriately modifying the objective function and non-negativity restrictions, we can rewrite the above LP problem into the form

$$\text{Max} \quad P = 40 x_1 + 30 x_2 + 0s_1 + 0s_2 + 0s_3$$

(2)
$$\begin{aligned}
\text{s.t.} \quad & x_1 + s_1 = 16 && \text{[cutting constraint]} \\
& x_2 + s_2 = 8 && \text{[mixing constraint]} \\
& x_1 + 2x_2 + s_3 = 24 && \text{[packaging constraint]} \\
& x_1, x_2, s_1, s_2, s_3 \geq 0
\end{aligned}$$

These three transformed constraints can also be expressed by the following matrix equation:

(3)
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 24 \end{bmatrix}$$

There are now five variables in all. The slack variables s_i are like x_1 and x_2 restricted to be non-negative. When $s_i > 0$, a slack exists in the ith constraint, when $s_i = 0$, the ith constraint is exactly satisfied. But s_i can never be negative. In the objective function, the slack variables are given zero coefficients because slacks do not contribute to profit.

It is an easy matter to find the values of the slacks implied by each extreme point. For the point (0, 0), for instance, we can substitute $x_1 = 0$ and $x_2 = 0$ into the three transformed constraints and find that $s_1 = 16$, $s_2 = 8$ and $s_3 = 24$. Thus the point $(x_1, x_2) = (0, 0)$ in the two-dimensional output space of above fig can be mapped into the point

$$(x, x_2, s_1, s_2, s_3) = (0, 0, 16, 8, 24)$$

in a five-dimensional solution space, as shown in the first row of the table below. By the same procedure, we can verify the mapping of the remaining four extreme points listed there.

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What is remarkable about the results of the table is the fact that all these five points in the solution space, which represent extreme points, share the common property that exactly three of the five variables assume nonzero values. This is no mere coincidence; besides, three is not just any number, but is the precise number of constraints in our LP.

Normally, assuming consistency and independence, we can expect a system of m equations to yield a determinate solution only when there are exactly m variables in it. In our present case, there are three constraints being considered in eq. (3), so that m = 3; consequently no more than three of the five variables can be included in the determinate solution with a nonzero value. (It may happen, however, that less than m variables actually take nonzero values in the solution space. This is known as the case of degeneracy.) This will perhaps

Output space (x_1, x_2)	Solution space $(x_1, x_2, s_1, s_2, s_3)$
(0, 0)	(0, 0, 16, 8, 24)
(16, 0)	(16, 0, 0, 8, 8)
(16, 4)	(16, 4, 0, 4, 0)
(8, 8)	(8, 8, 8, 0, 0)
(0, 8)	(0, 8, 16, 0, 8)

become clearer when we consider the following vector-equation version of the constraints:

$$(4) \qquad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} s_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} s_3 = \begin{bmatrix} 16 \\ 8 \\ 24 \end{bmatrix}$$

If we set any two of the five variables equal to zero, thereby in effect deleting two terms from the left of equation (4), there will result a system of three equations in three variables. A unique solution can then be obtained, provided that the (retained) coefficient vectors on the left are linearly independent. When the solution values of these three variables are combined with the arbitrarily assigned zero values of the other two variables, the result will be an ordered quintuple such as shown in the above table.

However, from this process, there can arise two possible situations. First, a negative solution value may appear; e.g., if we set $x_1 = s_3 = 0$, then we get $x_2 = 12$, $s_1 = 16$ and $s_2 = -4$. Because it is in violation of the non-negativity restrictions, such a solution is obviously unfeasible and must be rejected. Graphically, this particular unfeasible solution represents the intersection of the packaging border with the vertical axis, which clearly lies beyond the feasible region.

Second, all the solution values may turn out to be non-negative. In this eventuality, the solution will correspond to one of the extreme points of the feasible region, and the solution will constitute a *basic feasible solution* (BFS) of the LP. It is *feasible* because the solution is in the feasible region. It is *basic* because the availability of this solution is contingent on the presence of three linearly independent coefficient vectors, which together form a *basis* for a 3-space. In this way the search for the extreme points now amounts to the search for the basic feasible solutions of the transformed constraint equations.

As an illustration, let us set $x_1 = x_2 = 0$ in eq. (4). Then we have

$$(5) \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} s_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} s_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} s_3 = \begin{bmatrix} 16 \\ 8 \\ 24 \end{bmatrix}$$

or equivalently,

$$(6) \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 24 \end{bmatrix}$$

Since the leftmost matrix is an identity matrix that can be dropped without affecting the equation, the solution can be read off directly: $s_1 = 16$, $s_2 = 8$, $s_3 = 24$. The existence of this unique solution is guaranteed by the linear independence of the three coefficient (column)

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vectors, which are the unit vectors spanning a 3- space (the requirement space). Inasmuch as the solution is non-negative, it is BFS. And, since solution is on the point (0, 0) in the above fig, the BFS does indeed correspond to an extreme point.

Setting another pair of variables in (4) equal to zero will clearly result in an entirely new equation system. If the coefficient vectors are again linearly independent, we shall have a new basis for the requirement space, and a new BFS – provided the solution is nonnegative – which can locate for us another extreme point.

The significant thing about the BFS concept is that the method of its determination is algebraic instead of geometric, so that it permits a ready extension to the general LP with m constraints and n choice variables.

Finding the optimal extreme point: In a low-dimension program, we can of course find all the BFS's, calculate the corresponding values of the maximand (or minimand) and then selectthe optimal one therefrom. When many variables and constraints are involved, this procedure can entail too much computation. The idea of simplex method is to start with some initial extreme point, compute the value of the objective function, and then see whether the latter can be improved upon by moving to an adjacent extreme point. The process will continue till an extreme point is reached that does not admit of further improvement. This point constitutes the optimal solution. The technique is illustrated below.

From eqs. (5) and (6), we have already found an initial BFS by setting $x_1 = x_2 = 0$. That gives rise to the following quintuple in the solution space, S_1, and a corresponding profit figure, P_1:

$$S_1 = (0, 0, 16, 8, 24)$$
$$P_1 = 40(0) + 30(0) = 0$$

Simplex tableau: Tableau I

	P	x_1	x_2	s_1	s_2	s_3	Constant
Row 0	1	-40	-30	0	0	0	0
Row 1	0	1*	0	1	0	0	16
Row 2	0	0	1	0	1	0	8
Row 3	0	1	2	0	0	1	24

This same answer can also be obtained schematically by the use of what is called a *simplex tableau*, as shown in the table above.

In the table all the numbers appearing beneath each variable are the coefficients of that variable in the relevant equations, whereas the numbers in the constant column are those unattached to any variable. The vertical line to the left of the constant column is where the equals sign should be in the various equations. Thus, row 0 can be read as $P - 40 x_1 + 30 x_2 = 0$, which is simply a transposed version of our objective function. Similarly, row 1 states that $x_1 + s_1 = 16$ (the first constraint), etc.

To find a BFS is to find a particular *basis* for the three-dimensional requirement space. This means that, disregarding row 0 for the time being, we must pick out from the last three rows three linearly independent column vectors. Consequently, we can take s_1, s_2, and s_3 into the basis and accordingly set $x_1 = x_2 = 0$. If the x_1 an x_2 columns are (mentally) deleted from the tableau, the three bottom rows will reduce exactly to the form of eq.(6), giving us $(s_1, s_2, s_3) = (16, 8, 24)$, which is nonnegative and hence, a BFS.

Inasmuch as no output is produced, the profit is zero, and this BFS is obviously not optimal. In maximisation problems, it is always desirable (if available) to adopt as the initial BFS one that includes slack variables only, because they can automatically supply a ready-made basis.

The profit information can also be read directly from the tableau. For if row 0 is considered along with the three bottom rows, disregarding the x_1 and x_2 columns, the tableau translates into the equation system

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$$
(7) \qquad
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
P \\
s_1 \\
s_2 \\
s_3
\end{bmatrix}
=
\begin{bmatrix}
0 \\
16 \\
8 \\
24
\end{bmatrix}
$$

which reveals not only the slack values but also the additional information that $P = 0$.

Now to improve upon the profit, we have to switch to a new BFS by forming a new basis. The process, called *pivoting*, is to replace a column vector currently in the basis by another column vector that is currently excluded. For selection of the *outgoing* and *incoming* vectors (or variables) we have to turn to the objective function. The objective function indicates that the marginal profit rate of x_1 is 40 and that of x_2 is 30. It means the selection of x_1 as the incoming variable is more suitable as a profit booster. In terms of the tableau, the criterion is that we should select that variable which has the negative entry with the largest absolute value in row 0. Here the appropriate entry is -40, and the incoming variable is to be x_1. The x_1 column is called the *pivot column*.

The pivot column replaces one of the s_i columns, but this raises two problems. First, we have to decide which of the column is to go and second, we must see it that, as the new member of the basis, the pivot column is linearly independent of the old vectors that are retained. Both the problems can be solved, by transforming the pivot column into unit vector, with 1 in either of the three bottom rows, and 0s elsewhere. If the transformed pivot column has its unit element in row 1, so that it looks like the s_1 column, we can certainly let s_1 be outgoing variable, for this replacement will preserve the linear independence in the basis. Similarly, if the unit element is in row 2, we can let the transformed pivot column replace the s_2 column. In this way, above twin problems can be resolved at once.

Now, we have to decide on the exact placement of the unit element in the pivot column. The pivot-column element to be set equal to 1 is called *pivot element*. The overriding consideration in the choice of the pivot element is that we must stay within the capacity limitations. For this, to choose the pivot element, we have to – (1) pick out those elements in the pivot column, in all rows except row 0, that are positive; (2) divide each of thee positive elements into its counterpart in the constant column; (3) compare the resulting quotients, which we shall refer to as *displacement quotients*, and take the row with the smallest quotient as the *pivot row*; (4) and then select the element at the intersection of the pivot column and the pivot row as the pivot element. By selecting the smallest displacement quotient, we can make sure that the output added (x_1) will be small enough to stay within the bounds of all the constraints and that none of the variables will violate the non-negativity restrictions. In this example 16/1 is the smallest quotient and therefore row 1 should be the pivot row, with the asterisked element (1) as the pivot element. Our next task is to transform the pivot column into a unit vector by setting the pivot element equal to 1 and the other elements therein to 0s. If the pivot element is some number k, we can transform it to 1 by dividing all the elements in the pivot row, including the constant column, by k. The new version of the pivot row will then serve as the tool to transform the remaining rows into the desired form – with 0s in the pivot column. The results (and treatments) are shown in tableau II.

Tableau II

	P	x_1	x_2	s_1	s_2	s_3	Constant	Treatments
Row 0	1	0	-30	40	0	0	640	R0 + 40 R1
Row 1	0	1	0	1	0	0	16	-
Row 2	0	0	1	0	1	0	8	-
Row 3	0	0	2*	-1	0	1	8	R3 – R1

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Now the second basic feasible solution is:

S = (16, 0, 0, 8, 8)

P = 640

Since a negative entry in row 0 indicates a positive marginal profit rate, further profit improvement is possible by letting x_2 replace a zero-profit-rate or a negative-profit-rate variable in the basis. Therefore we take the x_2 column as the next pivot column and do the same process again to reach tableau III.

Tableau III

	P	x_1	x_2	s_1	s_2	s_3	Constant	Treatments
Row 0	1	0	0	25	0	15	760	R0 + 15 R1
Row 1	0	1	0	1	0	0	16	-
Row 2	0	0	0	1/2	1	-1/2	4	R2 – ½ R3
Row 3	0	0	1	-1/2	0	1/2	4	R3 – R1

Now the solution is:

S = (16, 4, 0, 4, 0)

P = 760

Here, since row 0 of tableau III contains no more negative entries, no further pivoting will prove profitable. To appreciate this fact, let us convert row 0 into the equation

$$P = 760 - 25s_1 - 15s_3$$

To maximise P in this equation, we must set $s_1 = s_2 = 0$; but the above solution does just that. Hence this solution must be optimal.

Steps for minimisation problems: In the maximisation problem, a ready-made initial BFS was available in the point of origin, because the point of origin normally lies in the feasible reason of a maximisation problem. But this is not possible for minimisation problems, because in this case the feasible region often does not include the point of origin. To illustrate we take the following example:

Example 5: Suppose to maintain good helth, a person must fulfil certain minimum daily requirements for three kinds of nutrients – calcium, protein and vitamin A. Also assume that the person's diet is to consist of only two food items I and II, whose price, nutrient contents and daily minimum requirement for each nutrient are as shown in table below. What combination of the two food items will satisfy the daily requirements at least cost?

	Food I (per kg)	Food II (per kg)	Minimum daily requirement
Price (in Rs.)	0.60	1.00	
Calcium (unit)	10	4	20
Protein (unit)	5	5	20
Vitamin A (unit)	2	6	12

Sol.: Let the quantities of the two food items purchased daily are x_1 and x_2. Then the problem can be stated as:

Min. Cost, $C = 6/10\ x_1 + x_2$

s.t. $10\ x_1 + 4\ x_2 \geq 20$ [calcium constraint]

(8) $5\ x_1 + 5\ x_2 \geq 20$ [protein constraint]

$2\ x_1 + 8\ x_2 \geq 12$ [vitamin A constraint]

$x_1, x_2 \geq 0$

Introducing surplus variables, s_i, above problem can be expressed as:

Min. Cost, $C = 6/10\ x_1 + x_2$

s.t. $10\ x_1 + 4\ x_2 - s_1 = 20$ [calcium constraint]

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(9) $5 x_1 + 5 x_2 - s_2 = 20$ [protein constraint]
               $2 x_1 + 8 x_2 - s_3 = 12$ [vitamin A constraint]
               $x_1, x_2, s_1, s_2, s_3 \geq 0$

Here, if we try to set $x_1 = x_2 = 0$, the three constraints will yield the solution $(s_1, s_2, s_3) = (-20, -20, -12)$, which is clearly unfeasible. Therefore we must now search for some suitable initial BFS.

One method that will simplify this search is to augment the program by adding a nonnegative artificial variable to each constraint. These variables, denoted by v_i, should be assigned some large coefficients in objective function, large in relation to the coefficient of the choice variables. If we let them be 100, then the program in (9) can be written as:

Min.   Cost, $C = 6/10\, x_1 + x_2 + 100\, (v_1 + v_2 + v_3)$
s.t. $10 x_1 + 4 x_2 - s_1 + v_1 = 20$ [calcium constraint]
(10) $5 x_1 + 5 x_2 - s_2 + v_2 = 20$ [protein constraint]
               $2 x_1 + 8 x_2 - s_3 + v_3 = 12$ [vitamin A constraint]
               $x_1, x_2, s_1, s_2, s_3, v_1, v_2, v_3 \geq 0$

Since the artificial variables are associated with three linearly independent unit vectors (which form an identity matrix), similar to the s_i variables in tableau I above we may accept the v_i variables into the initial basis. But, since these variables are artificial, they must be prevented from entering into the optimal solution. For this we should assign large coefficients to v_i variables in objective function. Since these coefficients (100 in this ex.) represent the prices of some artificially conceived food items, large values to these will automatically make these food items too ex expensive to be included in the optimal diet. This will then assure $v_1 = v_2 = v_3 = 0$ in the optimal solution.

The simplex method can now be applied in the same way, except for a few simple modifications. First, upon arranging the program in (10) into tableau, we see that the v_i column are not yet in the form of 4-element unit vectors. We first add $100 \times$ (row 1 + row 2 + row 3) to row 0, in order to convert tableau I into tableau II. Since the latter contains a 4×4 identity matrix, we can read off the solution $(C, v_1, v_2, v_3) = (5200, 20, 20, 12)$. Second modification is - for minimisation programs, we should look for the column, except column C and constant column, with the largest positive element in row 0. The subsequent steps to solution are shown in the table below:

Tableau	Row	C	x_1	x_2	s_1	s_2	s_3	v_1	v_2	v_3	Constant
I	0	1	-6/10	-1	0	0	0	-100	-100	-100	0
	1	0	10	4	-1	0	0	1	0	0	20
	2	0	5	5	0	-1	0	0	1	0	20
	3	0	2	6	0	0	-1	0	0	1	12
II	0	1	8497/5	1499	-100	-100	-100	0	0	0	5200
	1	0	10*	4	-1	0	0	1	0	0	20
	2	0	5	5	0	-1	0	0	1	0	20
	3	0	2	6	0	0	-1	0	0	1	12
III	0	1	0	20481/25	3497/50	-100	-100	-8497/50	0	0	9006/5
	1	0	1	2/5	-1/10	0	0	1/10	0	0	2
	2	0	0	3	1/2	-1	0	-1/2	1	0	10
	3	0	0	26/5*	1/5	0	-1	-1/5	0	1	8
IV	0	1	0	0	2498/65	-100	7481/130	-8998/65	0	-20481/130	35154/65
	1	0	1	0	-3/26	0	1/13	3/26	0	-1/13	18/13
	2	0	0	0	5/13	-1	15/26*	-5/13	1	-15/26	70/13
	3	0	0	1	1/26	0	-5/26	-1/26	0	5/26	20/13
V	0	1	0	0	1/15	-19/75	0	-1501/15	-7481/75	-100	56/15
	1	0	1	0	-1/6	2/15	0	1/6	-2/15	0	2/3
	2	0	0	0	2/3*	-	1	-2/3	26/15	-1	28/3

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						26/15						
	3	0	0	1	1/6	-1/3	0	-1/6	1/3	0	10/3
VI	0	1	0	0	0	-2/15	-1/10	-100	-2498/25	-999/10	14/5
	1	0	1	0	0	-3/10	1/4	0	3/10	-1/4	3
	2	0	0	0	1	-13/5	3/2	-1	13/5	-3/2	14
	3	0	0	1	0	1/10	-1/4	0	-1/10	1/4	1

When the cost is reduced to 14/5 in tableau VI, we can tell from row o that no further reduction is possible because no more positive entries are present in the x_j, s_i and v_i columns. Therefore the optimal solution is:

$(x_1, x_2, s_1, s_2, s_3) = (3, 1, 14, 0, 0)$ and C = Rs. 2.80

[Note: Introduction of the v_i variables lengthens the computation process. So we should try whenever possible to reduce the number of such variables added. This can be done, for instance, if the first column of the coefficient matrix happens to contain the elements 0, 1, and 0 (rather than 10, 5, and 2), for then we may omit v_2 and let x_1 take its place in the initial basis.]

Exercise

1. Maximise $P = 4x_1 + 3x_2$
s.t. $x_1 + x_2 \leq 4$
  $2x_1 + x_2 \leq 6$
  $x_1, x_2 \geq 0$ [*Ans.* $(P, x_1, x_2, s_1, s_2) = (14, 2, 2, 0, 0)$]

2. Maximise $P = 4x_1 + 3x_2$
s.t. $x_1 + x_2 \leq 5$
  $3x_1 + 2x_2 \leq 12$
  $x_1, x_2 \geq 0$

3. Maximise $P = 6x_1 + 2x_2 + 5x_3$
s.t. $2x_1 + 3x_2 + x_3 \leq 10$
  $x_1 + 2x_3 \leq 8$
  $x_1 + 2x_2 + 5x_3 \leq 19$
  $x_1, x_2, x_3 \geq 0$ [*Ans.* $(P, x_1, x_2, x_3 s_1, s_2, s_3) = (34, 4, 0, 2, 0, 0, 5)$]

4. Maximise $P = 3x_1 + 2x_2 + 6x_3$
s.t. $2x_1 + 3x_2 + x_3 \leq 10$
  $x_1 + 2x_3 \leq 8$
  $x_1 + 2x_2 + 5x_3 \leq 19$
  $x_1, x_2, x_3 \geq 0$

5. Minimise $C = x_1 + 4x_2$
s.t. $x_1 + 2x_2 \geq 8$
  $3x_1 + 2x_2 \geq 12$
  $x_1, x_2 \geq 0$ [*Ans.* $(C, x_1, x_2, s_1, s_2) = (8, 8, 0, 0, 12)$]

6. Minimise $C = 2x_1 + 7x_2$
s.t. $x_1 + 2x_2 \geq 8$
  $x_2 \geq 3$
  $x_1, x_2 \geq 0$

7. Minimise $C = 12x_1 + 42x_2$
s.t. $x_1 + 2x_2 \geq 3$
  $x_1 + 4x_2 \leq 4$
  $3x_1 + x_2 \leq 3$

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$x_1, x_2 \geq 0$ [*Ans.* (C, x_1, x_2, s_1, s_2, s_3) = (45, 2, 1/2, 0, 0, 7/2)]

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Derived Demand

The demand for a factor of production is derived demand. The demand for factors of production depends on the demand for the finished goods. The demand for lumber, paper, plywood etc. is not direct but is derived from the sale or expected sale of finished fabricated products: houses, furniture, newspapers, magazines, books, toys and hundreds of other goods that are made from them.

The concept can be explained with the help of the diagram given below. In the fig. D_{np} is a hypothetical demand curve for newsprint in Nepal and S_{af} the aggregate supply curve for all factors of newsprint production other than pulpwood, such as capital, chemicals, water, energy, labour etc.

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Recreation Valuation
The following methods are widely used for valuation of recreation:
1. Travel cost method (TCM)
2. Contingent valuation method (CVM)
3. Hedonic travel cost method (HTCM)
4. Unit day valuation method (UDV)

Travel Cost Method
A method to derive demand curves for recreation on the basis of travel cost incurred in visiting the site from different distances is the **travel cost method**. The approach uses simulated increase in travel costs to estimate a demand curve for visits. Recreation site demand curve is derived assuming that recreationists would react to increases in travel costs the same way they would react in increases in recreation area entry fees. The method can be illustrated by taking an example of a Picnic Park, where no entry fee is currently charged. The surrounding region is divided into three zones including all visitors according to the distance they have to travel to reach the Park. Table 1 shows hypothetical data on population, travel cost per visit from the midpoint of each zone to the Park, number of visits per year, and visits per 1000 population for each zone.

Table 1: Visitation data for Picnic Park

Zone	Population (thousands)	Travel cost per visit (Rs.)	No. of visits per year	Visits per 1000 population
1	2	10	600	300
2	1	20	200	200
3	4	30	400	100

In fig. 1, the number of visit per 1000 population in each zone is plotted against the travel cost per visit from Table 1, which can be referred as **trip demand curve.** This is not the demand curve for site, but it tells us how the visitation rate for each zone changes with a change in cost and so can be used to construct the site demand curve. To get the site demand curve, we will simulate increases in travel cost and assume that these would affect visitors the same as a park entry fee. Table 2 shows simulated increases in travel cost from Rs. 0 to Rs. 30 across the top due to the imposition of an entry fee and the visits from each zone at each fee level which is calculated on the basis of data in Table 1 and Fig. 1.
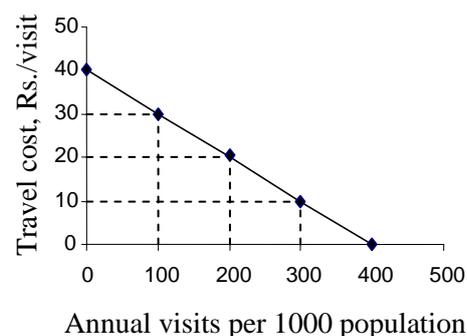
Annual visits per 1000 population
Fig. 1: Trip demand curve

Table 2: Total visits per year with simulated increases in travel costs

zone	Simulated increases in travel cost (as an entry fee)			
	Rs. 0	Rs. 10	Rs. 20	Rs. 30
1	600	400	200	0
2	200	100	0	0
3	400	0	0	0
Total visits	1200	500	200	0

Table 2 shows that at a zero price total visits are 1200, at price Rs. 10 it is 500 and so on. Plotting these points gives fig. 2, the estimated **site demand curve** for Picnic Park.

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We now present a more formal definition of the travel costs model. The underlying hypothesis is that number of visits to a site is inversely related to the cost of getting there.

i.e. $D_{ij} = f_i (TC_{ij})$ ………   (1)

and $f_i' < 0$

where, TC_{ij} = travel cost to site i for population j

   D_{ij} = annual number of visits to site i by members of population j

By assuming that tastes are similar for all population, travel is for use of site i only, and people respond to a entry fee in the same manner as an increase in travel cost, it is possible to derive the aggregate demand curve for a site. Equation (1) may be transformed to express visits from different populations as a per capita rate:

Fig. 2: Site demand curve

i.e. $D_{ij}/Pop_j = q_i (TC_{ij} + E_i)$ ……… (2)

where, E_i = entry fee to site i

   Pop_j = number of people in population j

Site use data can be used to provide values of D_{ij}/Pop_j, where the population refer to people living in zones with different travel costs to reach site i (TC_{ij}). E_i is zero in all cases, so the function q_i in equation (2) may be estimated. By varying E_i it is possible to estimate visitation rates, and hence the total visitors from each population at any given entry fee. Aggregation of visits across populations allows the aggregate site demand curve to be plotted.

In this model some weaknesses may be pointed out: population characteristics could differ among zones, travel experience may be positive or negative, travel time has a cost, users shift to competing sites when the price gets too high at one area, congestion can change park quality and trips may be very long and have multiple destinations. Many of these points are handled in more complex travel cost models. For example we can use following multiple regression equation for a trip demand function for one recreation area.

   Visits from zone Z = $V_z = c_0 + c_1T_z + c_2P_z + c_3I_z +$ ……… (3)

The c_j's are coefficients, T_z is cost of travel from zone Z to the park, P_z is population in zone Z and I_z is per capita income in zone Z. After estimating the coefficients with regression analysis, we can substitute the variable values into the equation and simulate increases in travel cost (the entry fee) and store the resulting visitation (V_z) for each fee. The same process can be done for each zone. Then for each increase in travel cost, we could sum the resulting visits across all zones to get total visits at each simulated entry fee.

More detailed travel cost models include things like multiple recreation sites with attractiveness indexes, more population characteristics, travel cost to substitute sites, nonlinear equations, multiple destinations etc.
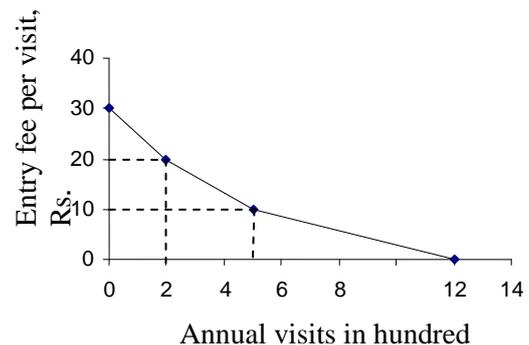
Contingent Valuation Methods

Another way to value non-market goods and services is to simply ask users what is the maximum they'd be willing to pay for them (*willingness to pay* - WTP) or what minimum compensation they'd require to willingly give them up (*willingness to sell* - WTS). This is the **contingent valuation method** (CVM), formerly known as *survey method*. The method uses either interviews, mail surveys or other survey techniques. A hypothetical market is constructed and questions are asked to find out consumers' WTP or WTS for some non-market goods or services. Typical questions would include: would you be willing to pay Rs. X to improve air/water quality, preserve views of attractive landscape?

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Since responses to a hypothetical situation are derived from potential consumers, CVM assumes that the consumer's expressed WTP or WTS in a hypothetical situation is a measure of the consumer's value in an actual situation. Similarly, in asking people their maximum WTP or WTS for some non-market good or service, gaming can be a problem unless questionnaires are carefully designed. For example, if respondents, say wilderness hikers, felt they wouldn't actually have to pay, they might cite very high values if they thought it would increase the chance of saving a wilderness area in jeopardy. But if users felt it was highly likely they'd eventually have to pay, they might cite low values to ensure low user fees on an area already earmarked for recreation.

However, if people understand clearly the change being offered and answer truthfully, CVM is ideal. It measures precisely what the analyst wants to know – the individual's strength of preference for the proposed change. Its principle drawback is that it depends on what people say rather than what they would do. Such biases in responses can, however, be minimised by careful survey design.

Another problem is which value should be used when WTP differs from WTS. Values of WTS can be much higher than WTP for the people with declining marginal utility of money. One approach is to use WTP in benefit/cost analyses of providing new non-market goods and to use WTS when estimating values of amenities lost or damaged.

Hedonic Travel Cost Model

The **hedonic travel cost model** focuses on valuing site characteristics. The approach estimates regression equations of travel costs, T_z, from each origin zone, z, as a function of several attributes, A_j, at several recreation sites. For example, with fishing areas, attributes might be acres of scenic areas, average size of fish, average daily catch per person, and acres of water. The equations, one for each origin, could take the form:

$$T_z = c_0 + c_1A_1 + c_2A_2 + c_3A_3 + \ldots\ldots$$

Since the A's are usually attractive attributes, the estimated coefficients, c_j, should have positive signs and are each origin's price of one added unit of each attribute. For example, if A_2 is acres of water, c_2 is the added willingness to pay travel costs (T_z) per added acre of water, or the per acre *hedonic price* of water for users from origin z.

With information from the above equations, for each attribute, analysts statistically estimate separate demand functions showing the amount of attribute consumed across all origins at different hedonic prices. The method is somehow complicated and requires very detailed data on site attributes and visitation rates and therefore it is not yet widely used.

Unit Day Valuation Method

In this method values of the non-market goods or services are set administratively. This method is used when budget limits use of TCM or CVM and where number of site visits is relatively small. An average WTP is estimated for recreation use activities on the basis of past experiences and consumers' surplus. A team of experts may be appointed for the purpose.

The accuracy of this method is highly suspected as being non-analytical and possibility of personal or political biasness.

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Non-market Valuation

Total economic value of a resource may be categorized as use value and non-use value. In the same manner value of non-market goods and services can also be categorised as use and non-use value. Use value may again be categorized as direct use value and indirect use value. **Direct use values** are values derived from direct use or interaction with a resource and its services. They involve both commercial, subsistence, recreation or other activities associated with a resource. For example: use of forest resource for timber, fuel-wood, NTFP, recreation and tourism etc.

Indirect use value relates to the indirect benefits which a person or society gets from a resource. For example, a forest provides following indirect benefits to the society: watershed protection, air pollution reduction, nutrient cycling etc.

Values like option value, existence value and bequest value are termed as non-use values. **Option value** is the willingness to pay for the option to use some resource in the future. For example, forest resources may be underutilized today but may have high future value in terms of scientific, educational, commercial and other economic uses. **Existence value** is a person's willingness to pay for the assurance that something will exist, even if we never plan to use or visit it. It is derived from the pure pleasure in something's existence, unrelated to whether the person concerned will ever be able to benefit directly or indirectly from it. For example, there are individuals who do not visit the Mount Everest but want its existence. **Bequest value** is the willingness to pay to preserve some resource for future generation. These apply mainly to unique resources for which there are few or no substitutes, for example, endangered species, certain old-growth forests, and unusual scenic resources.

Although use values are readily measured with TCM or CVM, non-use values can be measured only with the CVM. The more generalized a benefit become, the harder it is for people to respond to questions about willingness to pay. For example, respondents can more easily state how much they're willing to pay for a day hunting deer than for preserving scenic rivers or wild life reserves in their country.

In spite of the difficulties in measurement non-use value play an important role in making decisions by the authority. For example, imagine selling a unique area like Parsa Wildlife Reserve to private industry. Suppose they develop the reserve to establish large garment industry. There would no doubt be an international protest, and surveys could probably show that option, existence and bequest value would be more than enough, worldwide, to offset the development income. This means that preservation would be socially optimal. But inability to fully collect such values could often make preservation in a private market impossible. Or suppose projected use values aren't high enough to justify preventing hydroelectric dam construction in a unique canyon. Even minimal non-use values could be enough to change the decision to favouring preservation.

Forest Stumpage

Stumpage value is what buyers pay for standing timber ready for harvest. The emphasis is on immediate cutting. Investors' willingness to pay for timber to be cut in the future won't necessarily equal stumpage value. Stumpage value may be a price per unit volume, for example, a stumpage price in Rs. Per thousand board feet, per cord, or per cubic foot (cft). Stumpage value can also be expressed as value per unit area, for example, Rs. Per acre. Since stumpage income occurs in the very near future, stumpage valuation involves no discounting.

Now to explain the process of stumpage valuation let's take a simple example of a land area where timber is grown for lumber production only. Assume competitive conditions, one region, one species, and a given quality of lumber that mills sell for Rs. 1000 per cft. Suppose, this is a price that each mill must accept as a price taker and it costs a mill Rs. 400/cft,

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including profit, to process delivered logs into lumber and distribute it to buyers. Then such a mill would be willing to pay up to Rs. 1000 – 400 = Rs. 600/cft for logs delivered to the mill.

Suppose a logger can receive Rs. 600/cft for delivered logs, and it costs Rs. 200/cft for logging and hauling of a given type and location of stumpage (this include building logging roads, administration, felling, skidding, bucking, loading, hauling, and profit). That logger would be willing to pay up to Rs. 600 – 200 = Rs. 400/cft for such stumpage. In summary, this is shown below:

Lumber sale price:		Rs. 1000/cft
Cost of milling and distribution:		- 400
Delivered log price at mill:	Rs. 600	
Cost of logging and hauling:		- 200
Residual for stumpage:		Rs. 400/cft

Or, in general, given any end product like plywood or lumber:

Buyer's maximum willingness to pay for stumpage = (end product price) – (cost of manufacturing and distributing the end product) – (cost of logging and hauling logs) [*here cost also includes acceptable profits*]

Or, if the logger sells only delivered logs, the stumpage residual is log price minus logging and hauling cost. Assuming a given type and location of timber, this equation models stumpage valuation as a *residual value*, sometimes called **conversion return.** The seller may value stumpage similarly or could consider prices received by others on comparable sales; or for non-monetary reasons, the seller may set a very high minimum selling price. The rice below which an owner won't sell stumpage is the **reservation price.** Actual stumpage prices are set by bargaining between buyers and sellers, often with competitive bidding among stumpage buyers.

Above equation shows that, other things being equal, higher (lower) lumber prices lead to higher (lower) stumpage prices. The different processing costs at any level also affect the prices. This explains why, under competitive conditions, other things being equal, stumpage near mills tends to be worth more than stumpage far away. Similarly on steep land, higher costs of logging and road building will drive down stumpage prices.

Stumpage valuation often averages values for several species and log types such as veneer logs, sawlogs, poles, and pulpwood. Or standing timber may be valued differently for each log type and species.

Concept of Shadow Pricing and Shadow Wage Rate

The price mechanism operates imperfectly in underdeveloped countries. Market prices do not correctly reflect relative scarcities, benefits and costs. This is because perfect competition is entirely absent; structural changes do not respond to price changes; institutional factors distort the existence of equilibrium in the product, labour, capital and foreign exchange markets; and prices fail to reflect and transmit the direct and indirect influences on the supply side and demand side. "Market prices, particularly those of the factors of production, form a very imperfect guide to resource allocation in underdeveloped economies, because there exist fundamental disequilibria which are reflected in the existence of massive underemployment at present levels of wages, the deficiency of funds at prevailing interest rates and the shortage of foreign exchange at current rate of foreign exchange." [UN, ECAFE] In such a situation, the equilibrium level of wages would be much below the market wage rate, the equilibrium interest rates and the equilibrium rate of exchange would be higher than their market rates. In order to overcome these difficulties J. Tinbergen, H.B. Chenery and K.S. Kretchemer have emphasized the use of *shadow* or *accounting* prices.

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Shadow prices reflect intrinsic or true values for factors or products. J. Tinbergen defined them for the first time in 1954 in these words: "shadow prices are prices indicating the intrinsic or true value of a factor or product in the sense of equilibrium prices. These prices may be different for different time periods as well as geographically separate areas and various occupations (in the case of labour). They may deviate from market prices.

A UN report defines shadow prices in terms of the opportunity cost of the factor or product. the shadow price of an output such as capital, labour or foreign exchange represents its opportunity cost or the loss to the economy that would result from a reduction in its supply by one unit. A factor that is expected to be in short supply should have an accounting price higher than its market price, while one that is surplus should have a valuation that is lower than its market price. A. Quayyum defines shadow prices in terms of the marginal productivity of factors. E.J. Missan gives the simplest definition in these words: "a shadow or accounting price …… is the price the economist attributes to a good or factor on the argument that it is more appropriate for the purposes of economic calculation than its existing price if any."

Thus there is hardly any unanimity over defining accounting prices and the different concepts present so many difficulties in their calculation that the concept becomes ambiguous.

The concept and use of shadow price can be explained with an example of fixation of price for irrigation water, illustrated in the fig. Suppose, the government charge the price of irrigation water too low (OP_1) as part of its strategy of regional development. At this price demand for irrigation water will exceed its supply. In such a case the government may adopt the policy of rationing of water. It may ask each farmer to limit their land-area for irrigation. For this the government uses OP_s as the shadow price which is the equilibrium price when OQ_s of irrigation water is supplied and demanded.

Quantity of irrigation water
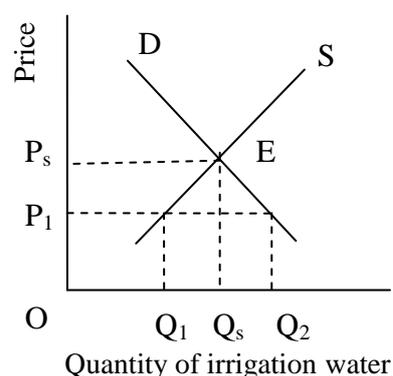
Determination of the shadow wage rate: The determination of the shadow wage rate is a difficult problem because labourers differ in efficiency. Therefore shadow price of labour cannot be the same for both the unskilled and skilled labour and for different types of skilled labour. In underdeveloped countries there is surplus labour in the rural areas having almost zero marginal product. But its shadow price cannot be assumed to be zero, it should be positive and provide a minimum subsistence level when such labour is employed on construction works. But even if the marginal product of labour is less than the wage, it does not necessarily follow that one should use a shadow price for labour lower than the wage. This is because wage earners tend to consume most or all the wages which they are paid. Thus the payment of wages constitutes real cost to the economy, even if there is no alternative employment for labour. Therefore, some economists are of the view that the accounting price for labour can be fixed anywhere above the zero marginal product of labour, and with the increase in the marginal product of labour its accounting price can also be raised. But, according to the UN experts, assuming no surplus of skilled labour but ample supplies of agricultural and unskilled labour the accounting prices of different kinds of skilled labour can be based on the cost of moving workers from villages to industrial areas, providing them with houses and other facilities, and training them.

Treatment of Time on Production

Individual consumers as well as societies make two types of economic decisions regarding production and consumption. The first type is of choice between one commodity and the other, between one activity and the other, the alternatives considered being at the same point

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of time. These are called **spatial decisions.** The other type is of choice between the same commodity or activity but at different points of time. These are called **temporal decisions.** If a firm decides to produce lumber rather than pulp from its supply of logs it makes a spatial decision. A firm makes a temporal decision when it decides to increase its outlay on silvicultural activities instead of paying higher dividends to its shareholders. These two types of decisions are not mutually exclusive. Often an economic agent makes decisions that have both spatial and temporal aspects.

All individuals and societies seem to value the same happenings occurring at different points in time in different ways. For example, a person would certainly prefer to receive Rs. 100 now rather than at some future date. But if he has to pay Rs. 100 then it is most likely that he would want to delay the payment as long as possible, if he is rational. It is the same with groups of people and with societies. Every society seems to want to consume now if it has an opportunity to choose between the present and future.

Future happenings are thus seen to be somehow diminished in their significance or discounted in the human psyche. There are many possible reasons behind this. One is that the future is uncertain. The other is that for monetary values, purchasing power of money diminishes due to inflation. But, even if all uncertainty and inflation were eliminated there still would be some preference for the present as compared to the future. By showing a preference for happenings close to the present rather than for those in the future we are acknowledging the value of time. Time preference between the present and future is the basis for making temporal economic decisions. Different individuals or societies discount future differently, i.e. the **rate of time preference** is different for different persons or societies.

Let us say a sum of Rs. 110 is payable to a person next year. If he is in need or impatient he will want to get the payment now, even if he gets a smaller amount than Rs. 110. Let us say he is willing to settle for Rs. 100 now instead of waiting to get Rs. 110 next year. This means he has discounted the value of Rs. 110 one year from now to Rs. 100 in the present. This discounting has meant dividing Rs. 110 by the factor 1.1. Therefore the discounting factor used by him is (1/1.1). This factor is expressed as 1/(1+0.1) and his rate of time preference is said to be 0.1 or 10% per annum. Sometimes this rate is also called the rate of discount.

The operation converse to discounting is called compounding. In the above example Rs. 100 now is compounded to Rs. 110 in one year. The relationship can be expressed, algebraically as follows:

$$V_n = V_0 (1 + i)^n$$

Where V_0 is the initial amount, V_n is the compounded sum after n years and i is the rate of interest per year.

While a certain degree of impatience is inherent in human nature and there is always a desire to consume now rather than later, there is also an opposite drive of a similar nature. People do want to consume later also. They do not have an infinitely high rate of time preference which would reduce the significance of all future happenings, as seen today, to nothingness. The productive processes in society use resources in the present so that consumption is made possible in the future. The process of sacrificing some of the present consumption for the sake of the possibility of more consumption in the future is called saving and investment or capital formation. In all modern economies investment activity is undertaken by economic agents because it offers the possibility of providing more in the future by sacrificing some in the present. The more productive an investment possibility is, the more investment is likely to be made in it.

Both individuals and societies operate under the influence of two very powerful economic forces. The first is the impatience regarding consumption which drives them to discount future happenings. The other is the productive possibility of investment which entices them to deny consumption now, i.e. to save and then invest so as to get compounded sum in future. Each

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society or individual comes to its own equilibrium between the pulls of these forces through the discount rate.

Social Discount Rate

The **social discount rate** is the premium which the society puts for preferring the present consumption to future consumption. This is explained in the fig. given below where the present consumption is taken on x-axis and future consumption on y-axis. C_1C_2 is the **transformation frontier** or the **investment possibility curve**, which is depicted on the basis that if the society sacrifice some of its present consumption, i.e. save and invest, it will produce more for future consumption. The fig. shows that if the society chooses to consume OC_1 in period t_1, then it will have nothing in period t_2. However, if all possible consumption, i.e. OC_1 is saved and invested in period t_1, it can get OC_2 in the next time period t_2. There are numerous other possibilities that the society can choose by staying at any point on the curve C_1C_2. For example, by being at point P society can consume OA in period t_1, save and invest amount AC_1, and thus have OB available to it in period t_2. An investment of AC_1 in period t_1 results in the production of OB in period t_2. The slope of C_1C_2 is the **marginal rate of transformation** of consumption from one period to another. As this transformation is possible through investment, the slope of the curve C_1C_2 is described here as the **marginal productivity of capital** (MPC_{12}).
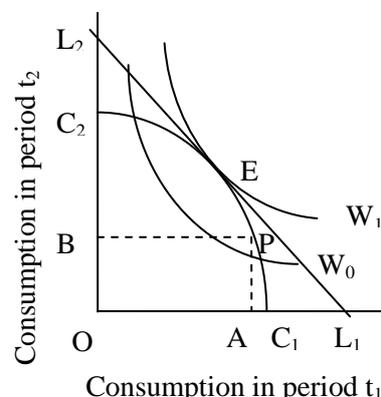
Similarly, the inter-temporal choices of consumption, i.e. the different combinations of consumption between the current and future period which would result in a given level of utility to the society are represented by the indifference curves W_0, W_1 etc., called social indifference curves. These curves show the time preference of the society between present and future consumption. The slope of these curves are called the **marginal rate of social time preference** ($MTPS_{12}$), which really is the marginal rate of substitution between consumption in the current time period and in a future period.

Now, given the fact that this society cannot go beyond the consumption possibilities shown on C_1C_2, the highest level of utility that can be obtained is by being at the point E. At this socially optimal point the slopes of C_1C_2 and W_1 are equal to the slope of the line L_1L_2. This common slope is a measure of the marginal social rate of time preference, as well as the marginal social productivity of capital. Algebraically this can be expressed as:

$$MTPS_{12} = MPC_{12} = - OL_2/OL_1$$

The ratio OL_1/OL_2 can be written as $(1+r)$ if r represents the fraction by which the investment in period t_1 grows to become the consumption possibilities in period t_2. In other words

$$r = (OL_2 - OL_1)/OL_1$$

The rate r is both the rate at which investments grow, at the margin, from one period to the other and the rate, at the margin, at which society discounts future consumption to make it comparable to consumption in an earlier period. In the former sense r is a measure of the **marginal rate of return on capital** or the **social opportunity cost of capital** or simply the **market rate of interest**. In the latter sense it is a measure of the **marginal social rate of time preference** or simply the **social rate of time preference** (SRTP) or the **social discount rate** (SDR).

Difficulties in actual determination of social discount rate: If there exists an economy in which all consumers and producers of the current and future time periods could represent their interests in the market place, and if there were perfect capital and investment markets without

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externalities, then, as already indicated, the market rate of interest in such a competitive economy would be the SRTP or SDR. However, in the real world a perfect market economy does not exist. Imperfect capital and investment markets are common place. In addition to imperfect capital market, consumers and producers of the future periods can not be correctly represented by the present consumers and producers. Markets, thus fail to indicate the optimal social discount rate.

In addition selection of a discount rate for economic analysis depends upon the agency (private or public) in whose interest the analysis is being carried out. For the purpose of a private agency, the rate of discount chosen may be the opportunity cost of capital (OCC), i.e. the interest at which funds can be borrowed in the market, however, the choice of a correct value for the social discount rate is a much more controversial and sensitive issue.

Economists generally agree that under the existing market imperfections the best that can be done is that the SRTP be decided by the government. The decision would be reliable only if the government were enlightened enough to consider such issues as the present and future, externalities, public goods, income distribution and goals of society fully before coming up with its figure. The next best alternative to an enlightened government decision is the choice of a rate that reflects the **social OCC.**

The value of social OCC can be determined from the private OCC only to the extent that the former must be lower than the latter. Perhaps the interest rate closest to the social OCC in the market is the interest earned by the government on long term bonds.

When future values are in current rupee the nominal rate is the appropriate one to use. However, when future values are expressed in constant rupee the real rate should be used. The difference between the two is due to the rate of inflation which can be computed from the consumer price index. The real rate of social discount, r, is obtained by adjusting the nominal rate, I, for the rate of inflation, p. The relationship can be written as follows:

$$1 + r = (1 + i)/(1 + p)$$

or

$$r = (1 + i)/(1 + p) - 1$$

Market Interest Rate

Capital in monetary form is an extremely unusable commodity. It has its own earning power. Its use over a period of time is worth something to the user and owner expects some return for it. Interest is the rental price of money, the reward for waiting. It is the use value of money and the price paid for the loan, i.e. use of a specified sum of money for a specified period of time. When money is borrowed, a price is set for its use which is termed interest. The rate employed gives a measure of importance to the time element involved.

The market rate of interest is the rate at which an investor actually pays to the lender for the use of money so borrowed or the lender actually receives from the borrower for making the sum available to him for a specific period of time. The market rate of interest reflects the opportunity cost of capital. Generally, there are at least two interest rates in a market, a lending and a borrowing rate. The borrowing rate is usually higher than the lending rate. The main reason for this difference is the administrative expenses of the organisations involved in this business. Sometime there are various lending and borrowing rates in a market. This is mainly because of the market imperfection and risk factor involved when money is lent. If the risk factor is excluded from the market rate of interest, it is often called pure or economic rate of interest.

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Project Formulation and Evaluation

Project: A project is a systematic plans or scheme of actions for achieving some objective/s. It involves a number of activities or programs which have to be taken up for meeting the desired objective/s. Most projects are inter-related. This refers to a situation wherein one scheme does not fulfil its purpose/objective without the other.

The process of designing a project consists of rejecting and accepting alternatives for executing a job (which is constituent of the project) and evaluating each phase by applying the same principle. The best alternative for each of the numerous jobs needed to achieve the objective/s are then combined to form the project.

Stages of Project Formulation

Pre-investment Stage:

i) **Identification:** The first important task in product formulation is to identify with precision the problem, objectives or goals of the project. This will in turn depend on regional/state/national/international priorities. The area is which the project is to function is identified. This is based on following aspect: (a) the need for the project in the area, (b) availability of required inputs in the area, (c) backwardness of the area, etc.

ii) **Preparation and feasibility:** Feasibility study are done on the following aspect –

a) Market: search for the market of the product or service so produced by the project

b) Technical aspect: whether the project is technically possible to conduct

c) Financial aspect: estimation of financial viability of the project on the basis of cost and probable benefit. Financial analysis is taken from the perspective of the private investor who is typically interested in the actual money costs and returns on his project.

d) Economic aspect: estimation of net benefit to the society by including both the direct and indirect cost and benefit to the whole society/economy. Economic analysis goes beyond the financial analysis in order to perceive the economic costs and benefits of a project on the welfare of society as a whole.

e) Operational aspect: whether it is possible to operate the project with regard to – impact on the ecology and environment, local socio-economic impact, financial implications, availability of human resources

iii) **Appraisal:** Appraisal is a process in which it's likely success or otherwise is evaluated before actual implementation. Project appraisal is done in a way, that the following virtues are examined:

a) impact on the local and regional socio-economy,

b) contribution to development,

c) justification for use of scares land, human, financial and other resources,

d) generation of employment,

e) impact on the environment,

f) likely outputs and also the term over which they would be available,

g) cost effectiveness analysis,

h) benefit – cost analysis.

Cost effectiveness analysis: Cost effectiveness is a criterion for determining the most efficient (economical or cost effective) way of achieving a specified objective. Cost effectiveness analysis (CEA) is used when the major benefits of a project prove exceptionally difficult to quantify, or where an environmental goal has been set by national authorities or an international agreement.

The first step in the CEA is to fix a target, expressed in non-monetary units. For example, the analyst may be asked to determine the most economical way to ensure a certain level of species diversity. Provided that policy makers have determined the minimum acceptable level

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of species diversity, it is relatively simple to select the form/s of land use which preserves that level of species diversity while generating the highest economic return.

CEA consists of calculating all the costs, both capital and recurrent, of a project, applying the appropriate shadow prices, and discounting the resulting stream to obtain a present value for costs. This procedure is repeated for the main alternative ways of carrying out the project, and one with the lowest present value is chosen. This criterion assumes that all the alternatives being compared can carry out the project equally well. If there are quality differences in the service being supplied, then the basis for the comparison is invalid. This type of analysis is generally used for investments in production of public goods/services like national defence, medical and health programs, atomic research etc.

Benefit – cost analysis (BCA): BCA is the most common method of economic project and policy appraisal. BCA is a decision tool which judges projects according to a comparison between their costs (disadvantages) and benefits (advantages). This type of analysis is generally applied to natural resource development or other development projects. Following four methods are generally adopted for BCA:

1) Net present value (NPV): NPV of a project is the present value of benefits minus the present value of its costs. Present value of benefits and costs is computed by discounting a set of benefits and costs that occur through time back to the beginning of the base year ($t = 0$).

$$\text{NPV} = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} \qquad \text{or} \qquad \text{NPV} = \sum_{t=0}^{n} \frac{(B_t - C_t)}{(1+i)^t}$$

Where, B_t = cash receipt or benefit at any year t
C_t = investment or cost at any year t
i = the discount rate
n = number of years

According to NPV guideline, a project is accepted if NPV is zero or greater. Project with a negative NPV are unacceptable.

2) Internal rate of return (IRR): IRR is defined as that discount rate at which the present value of benefits equals the present value of costs, i.e. the net present value is zero.

$$\sum_{t=0}^{n} \frac{B_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} = 0 \qquad \text{or} \qquad \sum_{t=0}^{n} \frac{(B_t - C_t)}{(1+i)^t} = 0$$

Here i is the internal rate of return. It is usually found by trial and error method.

The IRR investment guideline says that a project is acceptable if its IRR is equal to or greater than the minimum acceptable rate of return (MAR). Projects with IRR < MAR are unacceptable.

3) Benefit /cost ratio(B/C): A project's B/C ratio is the present value of benefits divided by the present value of costs.

$$\text{B/C ratio} = \sum_{t=0}^{n} \frac{B_t}{(1+i)^t} \div \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}$$

According to the B/C ratio criterion, projects are acceptable when the B/C ratio is 1 or greater than 1 and unacceptable if B/C ratio is lesser than 1.

4) Payback period (PBP): The payback period is the number of years it takes to recover the cash investment in the project. It is the initial investment divided by the annual cash flow:

PBP = N/a

Where, N = the total initial investment
a = the annual cash flow

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According to this criterion that project is selected which has shortest payback period. Though the ratio is simple to calculate and easy to understand, it has several disadvantages. First, it can give the wrong answer if the cash flows are uneven each year. Second, the time value of money is not considered. Despite its shortcomings, PBP is often useful if considered together with measures such as NPV and IRR. This method is more useful in private projects.

**iv)	Financing:** Availability and arrangements of funds is one of the most important parts of project formulation. It is necessary that funds are available as and when required. It is not enough to ensure adequacy of long term capital resources. The success of the project also depends on ready availability of short term or working capital resources. Major sources of finance for forestry or other development projects are: central and local government, credit from international agencies like World Bank, ADB etc., aid from international donor agencies and foreign countries etc.

Investment and Design

Design implementation: This includes – (i) preparing for time schedules for all jobs so that the physical and financial targets of the project are phases appropriately. (ii) Distributing works to various departments or agencies having the appropriate technical expertise.

Post Investment Stage

Operation and expert evaluation: This includes - (i) Executing and controlling the project which requires frequent reviewing, updating the activities and constant action to restore the operation to its planned characteristics. (ii) Evaluating the performance of each project/program involving the experts of the related field to ensure the worth of good and service for each rupee spent.

Project Evaluation

Use of BCA: (i) It helps in reducing differences in the marginal effectiveness of alternative measures for accomplishing such objectives as between irrigation and other means of raising agriculture production.

(ii) It helps in assessing the costs of fulfilling one objective in terms of benefits sacrificed with respect to others.

(iii) It has a political advantage in that, it would be difficult for any particular group to distort project plans to serve its own interests.

(iv) It permits decentralized decision making. Even if the public sector is small, no single authority can hope to handle the vast mass of technical information needed to decide on a number of specific projects. In order to calculate costs and benefits of each project, a separate authority is needed for each. This, therefore, necessitates decentralization of decision making.

(v) It is a practical way of assessing the desirability of projects, where it is important to take a long view (in the sense of looking at repercussions in the future, as well as the nearer future) and a wide view (in the sense of allowing for side effects of many kinds on many persons, society, region etc.)

Limitations of BCA:

(i)	Difficulties in cost assessment: mainly due to market imperfection, use of shadow prices.

(ii)	Difficulties in benefit assessment: due to uncertainty regarding future price, demand and supply.

(iii)	Arbitrary discount rate: social rate of discount is likely to be arbitrary.

(iv)	Neglects joint benefits and costs.

(v)	Ignores opportunity costs.

(vi)	Adjustment for risk and uncertainty.

(vii)	The problem of externalities: difficult to measure.